Measuring Signal-to-Noise Ratios in MR Imaging

The signal-to-noise ratio (S/N) in magnetic resonance imaging is one of the variables that must be measured when comparing the relative performance of different techniques. Although various investigators and official groups have proposed different methods for measuring S/N, these are generally not practical for use by a physician working in a clinical situation. The authors present a simple method that should serve for estimating S/N in most cases.

Index terms: Magnetic resonance (MR), image processing • Magnetic resonance (MR), physics • Magnetic resonance (MR), technology

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There exist a wide variety of reasons for measuring and reporting signal-to-noise ratio (S/N) in magnetic resonance (MR) images. A typical use found in the pages of this journal, and one that should be encouraged, is for comparing the relative performance of different imaging techniques in clinical investigations. Various investigators and official groups have proposed different methods for measuring S/N, but these are not practical for use by a physician working with patients.

Noise in an image is, in its most general definition, any variation that represents a deviation from truth. Noise sources can be systematic (so that repeating the procedure will show the same effect) or random. Nonrepetitive noise (such as that resulting from motion) can be structured or can follow some sort of statistical distribution. Structured noise is an important component of diagnostic images (1-4), but its treatment is beyond the scope of this article. The distribution of statistical noise can be affected by data-processing algorithms that smooth and/or enhance the edges and by nonlinear intensity assignments, among other factors. For the purpose of this article we will assume that all processing is linear and that if some degree of smoothing exists, it is relatively small.

Generally, for the purpose of evaluating an imaging technique, the variable of interest is how much a single pixel deviates from some "true value," generally defined by the mean value of all the other pixels expected to have the same value (for instance, all the pixels representing normal liver). When working with a uniform phantom, a simple calculation of standard deviation (SD) should yield such a value; in an actual experiment, however, it is unlikely to do so. This SD may be affected by small nonuniformities over the phantom, such as those resulting from background, gradient, or radio-frequency field nonuniformities. To reduce the effect of large-scale (i.e., long-spatial-wavelength) disturbance, the SD can be calculated by measuring the difference in signal level between each pixel and its eight immediate neighbors. The major source of error can come from ringing due to finite sampling of the object. This can be minimized by making sure that the phantom does not have hard, well-defined edges. For instance, if a conical phantom is imaged across its axis, with angles such that within a section the edge would blur over a few pixels, then the disturbances from sampling errors would be minimized adequately.

Nevertheless, the relevant conditions, especially the condition of finding a uniform region over which to measure noise, are not always easily met in a patient. As a consequence, it is natural to look for noise where there is no signal, that is, over air. It is here that pitfalls can occur due to the various ways in which MR data are handled.

WHERE TO MEASURE NOISE

In general, the image plane has two distinct axes. One is along the readout direction, and the other is along the phase-encoded direction. MR units use filters to limit the frequency range of the acquired signals in the readout direction to that of the field of view of interest. These filters typically "roll off" the signal (and the accompanying noise) over a finite distance (i.e., over a finite range of frequencies). If the object fills the field of view defined by the filter

Abbreviations: FT = Fourier transform, SD = standard deviation, S/N = signal-to-noise ratio, TR = repetition time.
and the filter fills the displayed field of view, then there will be no region where noise can be measured. Potentially misleading is the case in which the filter cuts off information at the edge of the object, so that its presence is not obvious. If the mean signal in air is measured along the readout axis in a region outside the filter, the air signal will be very small because of the effect of the filter, and the S/N will be artificially elevated. Depending on how the object fills the field of view, it may be difficult to assess whether one is operating outside the filter region. The surest way to ascertain where the filter cuts off, in this case, is to obtain an image with the same variables but without an object in the imager. The regions where the filter cuts off (if they are within the field of view) will be seen as two low-intensity bands at the edges of the field. This problem does not exist along the phase-encoded axis. On the other hand, any inconsistencies in the data (such as those that result from patient motion, field drift, and gradient power supply nonlinearities) will result in signal “bleeding” along the phase-encoded axis. Use of a measurement obtained there will lead to overestimation of noise from statistical sources but may well be a valid measure for comparison purposes if one is interested in assessing the impact of various techniques on image quality, including resistance to motion artifacts as a criterion. In general, though, the air signal should be measured outside the object, along the readout axis, and away from the region of filter cutoff.

**WHAT TO MEASURE**

Before deciding what quantity to measure in air, we need to know what sort of reconstruction has been carried out and how the data are stored in the image. The most common reconstruction in use produces a magnitude image. Two channels of time-domain data are acquired: the real channel and the imaginary channel. After a Fourier transform (FT), the signal magnitude $M$ is computed as $M = \sqrt{R^2 + I^2}$, where $R =$ real channel and $I =$ imaginary channel. Let us see what happens to noise in this case. The original time-domain data consist of positive and negative values of signal on which noise is superimposed. If there is no signal, the noise will have a distribution around zero. (This is the case unless there is a DC offset in the signal, in which case the noise will be distributed around DC. If left uncorrected or unadjusted, DC appears at a single point in the center of the field of view and need not concern us further.) The FT of the signal plus noise can be considered as the FT of the signal plus the FT of the noise separately. The FT of the noise component of the two time-domain channels results in real and imaginary channels with noise also distributed around zero. Consider now what happens when we compute $R^2$ and $I^2$. Let us think of a 10-point case with the values $51, -5, 16, -7, -41, 86, -43, 40, -14$, and $-83$ in the real channel and $-1, -28, 61, -4, -86, 94, -46, -11, 21$, and $0$ in the imaginary channel. The mean value for each is zero, and the SD is 48. The SD is the noise in the data, which we will call SD of the object (SD$_o$). When we obtain the signal magnitude, the 10 data points become $51, 28, 63, 8, 95, 127, 62, 41, 26$, and $83$, respectively. This set has a mean of 58 and an SD of 34. We can see that obtaining the signal magnitude reduces the SD of the resulting signal in air (SD$_a$), although the noise is the same in an area with object signal. This can be ascertained from adding a signal value of, for example, 500 to the real channel value. The mean magnitude is 502, and the SD is 48. This apparent reduction of SD in air is due to the rectification process. As an extreme, consider a data set in which all the values are between $+1$ and $-1$. The range of values is then 2. If we rectify these data (by taking the square root of the square of each point), then all the values will be between $+1$ and zero, and the range will have been halved. This is roughly what happened in the previous example, in which the SD$_o$ only is reduced by a factor of two. When a rigorous analysis is done on normally distributed noise (5), what we find is that the mean value of the signal magnitude in air approximates the SD$_o$. To be precise, the mean value in air is $1.253 \times$ SD$_o$, that is, use of the mean value in air will result in overestimation of noise (and underestimation of S/N) by about 25%. The SD of the signal magnitude in air is about $0.655 \times$ SD$_o$, that is, its use will result in underestimation of noise by a factor of 1.53. For the purpose of reporting S/N in clinical evaluations, it is our suggestion that when the reconstruction produces a magnitude image, the signal should be measured as the mean in a region of interest of the subject and the noise should be measured as the mean in an appropriate region outside the subject, and these values should be reported as such. If the investigator chooses to report the more rigorous value, such that S/N is increased by 25%, this is acceptable, but it should be so noted.

Another reconstruction method is that which produces phase-sensitive images, or images of the real component of the data. In this case there is no computation of magnitude and its consequent noise rectification. The noise remains distributed around zero, and since a negative number cannot be explicitly displayed in a monochrome monitor, one of three things is done: (a) Only positive values are kept and displayed. In this case, only half the noise points are preserved. In our previous example, the 10 points in the real channel become $51, 0, 16, 0, 0, 86, 0, 40, 0, 0$, with a mean of 21 and an SD of 35. Both the mean and the SD are smaller than the SD$_o$. A rigorous analysis shows the mean to be $0.40 \times$ SD$_o$ and the SD to be $0.58 \times$ SD$_o$. (b) A constant is added to the result. In that case, the SD$_a$ is the SD$_o$. (c) Only positive numbers are displayed, but negative numbers are kept in memory. In this case, the SD$_a$ is the SD$_o$. Clearly, an awareness of the exact manipulations to which the data have been subjected is needed before deciding on a measuring method.

**HOW TO COMPENSATE**

For S/N comparison purposes, it is often necessary to normalize different acquisition variables. Some of these are repetition time (TR), section thickness, spatial resolution (voxel volume, or V), number of phase-encoded steps (N), number of excitations per step (n), and the derived variable time ($t = N \cdot n \cdot TR$) in two-dimensional FT. For three-dimensional FT, for S sections, $t = N \cdot n \cdot S \cdot TR$. To compare different techniques, it is sufficient to remember that S/N is proportional to V, to $\sqrt{(nN)}$ in two-dimensional FT, to $\sqrt{(nN/S)}$ in three-dimensional FT, and to $\sqrt{t}$ in both. Consequently, if all the other primary variables listed above are kept constant, a 10-mm section thickness will have twice the S/N of a 5-mm section. A 1 X 1-mm in-plane pixel (dimension at acquisition) will have twice the S/N of a 0.7 X 0.7-mm in-plane pixel; an image with 256 phase-encoded steps will have 1.15, 1.41, 1.63, and 2 times the S/N of images with 192, 128, 96, and 64 phase-encoded steps, respectively; and an acquisition with $n = 4$ will have 1.41 times the S/N of one with
In three-dimensional FT, if the other variables listed above are held constant, a sequence of 32 sections with \( n = 1 \) has the same acquisition time and S/N as a sequence of 16 sections with \( n = 2 \) but with twice the coverage (along the section axis) at constant section thickness. If a comparison of S/N per unit time is needed, at constant \( V \) (or corrected for \( V \)), it suffices to normalize the data to \( \sqrt{t} \). Sometimes, when contrast is not of importance, we may wish to compare S/N per unit time for techniques of different TR. Since for constant time TR can be traded off for signal averaging or for number of phase-encoded steps, TR can be treated similarly, so that the data must be normalized similarly, so that the data must be normalized to \( \sqrt{t} \). Note that since \( t = n \cdot N \cdot TR \), once again we are saying that S/N per unit time is normalized by \( \sqrt{t} \).

Thus far, we have considered variables “visible” to the operator. A less visible variable is the sampling interval, or the time between each sampled data point in the echo. In the above analysis, sampling interval was assumed to be constant. The longer this time, the higher the S/N. It is beyond the scope of this article to provide a full treatment of sampling interval, which we must assume is as long as possible given sequencing, artifact, and field strength constraints.

CONCLUSION

S/N is one of the variables that must be measured when comparing imaging procedures. While it is usually obvious how to measure signal, it is not immediately obvious how a reliable measure of noise can be obtained in clinical work. In this article, we give some suggestions that should serve in most cases. For situations that range outside those covered here, it is hoped that we have given the reader some guide to the relevant questions that need to be asked so that meaningful measurements can be made.

Finally, we note that S/N is not the ultimate determinant of diagnostic quality. Contrast and freedom from artifacts are as important as S/N, if not more so. Manufacturers already strive to improve S/N at the expense of other facets of image quality, and this may worsen if S/N reporting becomes common. As a reviewer of this article stated: “We will just have to live with that.”

References